LATERAL TRANSHIPMENT WITH CUSTOMER SWITCHING

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ABSTRACT

We consider the inventory transshipment problem between two retailers that sell the same product to two independent markets. One assumption that distinguishes our work is that unsatisfied customers may also switch to the retailer with surplus inventory. We examine the impact of such customer switching behavior on firms' inventory decisions and profits. We identify situations when the firm with surplus inventory is willing to fully transship, partially transship, or not transship. We show that there is a unique Nash Equilibrium in inventory decisions under certain conditions, and compare the case with central coordination with decentralized decision making.

INTRODUCTION AND LITERATURE REVIEW

When a customer walks into a retail store and finds the product she intended to purchase unavailable, both the retailer and the customer may take action. The retailer may request transhipment from a partner store where the product is available. If the retailer is not able to arrange the transhipment, the customer may turn to the partner store and buy the product by herself. The interaction of these two strategies and its impact on retailers' inventory decisions are interesting to study.

Inventory transhipment between retailers has been a common strategy when stockout occurs at one location and inventory surplus exists at the other location. Such strategy can be viewed as a way of virtually centralizing stock and taking advantage of inventory pooling. Therefore, when transhipment decisions are made under a central coordination scheme, it increases profit. However, if such a central coordination scheme does not exist, transhipment decisions are made locally for the interest of each individual retailer. As pointed out by Rudi et al. [4], such interfirm transhipment, rather than intrafirm transhipment, changes firms' inventory decisions since the risks imposed upon retailers are altered.

On the other hand, customer switching has been a well-observed phenomenon. If two retailers carry the same products, customers first go to their preferred retailer due to factors such as price, service, location, promotion, convenience, etc. Since the two retailers differentiate themselves in these aspects, they each have their own customer demand and do not compete directly. However, if customers cannot find the product in their preferred retailer, they may look for the product at another retailer. Not all customers do so since some of them may decide not to buy the product at all or buy a substitutable product. Such customer switching behavior is called “market search” in Anupindi and Bassok [2], where they assume that a fixed fraction of the customers will search for the goods at another retailer. However, since firms may not obtain the exact information on the proportion of customers who switch to another retailer, we consider in this paper that a random fraction of those will switch.

While making inventory purchasing decisions, it is important that firms take into consideration both transhipment agreement and customer switching behavior, since they interact with each other. In this
paper, we examine two situations. In the centralized case, the two retailers are coordinated to maximize total profits. In the decentralized case, each retailer maximizes its own profit. In each case, we examine the optimal inventory purchasing decisions and transhipment decisions and aim at answering following questions:

- What is the optimal transhipment strategy for the firm with surplus stock? How much inventory to reserve and how much to tranship? What factors influence such decisions?
- In centralized case, what is the optimal inventory purchasing strategy? In decentralized case, is there a unique Nash Equilibrium for inventory purchasing quantities?
- How do the decisions in decentralized case compare to centralized case? Shall the firms order more or less inventory? Shall the firm with surplus reserve more or less inventory?
- Is there a coordination transhipment price such that centralized profit can be achieved under decentralized setting?
- What is the impact of customer switching behavior on inventory decisions, transhipment decisions, and profits? Should such switching behavior be encouraged?

Our project is closely related to the literature focusing on lateral transshipment in a decentralized system. The typical literature study the equilibrium behavior, and check whether it is possible to design a mechanism to coordinate those independent partners to achieve the highest possible supply chain benefit. Rudi et al. [4] and Hu et al. [3] show the uniqueness of Nash equilibrium in the order quantities for two retailers, and discuss the existence of the coordinating transshipment prices. Anupindi et al. [1] study the cooperative game in transshipment stage but non-cooperative game in order stage. A common assumption shared by these papers is that one partner's demand never switches to the other partner even if the local market stocks out but the other market still has inventory. Hence, if there is no transshipment between them, each partner's market is completely separated from the other's, and there is no competition on attracting demand. While in our paper, we assume that consumers in one market do "market search" by themselves and go to buy from the "remote" market when they find their local market is out of stock.

**MODEL ASSUMPTIONS**

We consider a one-period model where two retailers i, j= 1, 2 sell to their independent markets with random demand $D_i$, $D_j$, respectively. We label $D_i$, $D_j$ as "local demand". Let $G_i(.)$ and $g_i(.)$ be the cdf and pdf for $D_i$. We assume that $G_i$ and $G_j$ are twice differentiable and strictly increasing on their supports. All the parameters for firm i and j can be asymmetric. All the assumptions below for firm i applies correspondingly to firm j.

Our model differs from Rudi et al. [4] by assuming that a random fraction of the unfulfilled demand after transhipment can switch to buy from another firm. The events take place as follows. Before observing demand, retailer i decides to purchase inventory $Q_i$. Simultaneously, firm j decides to purchase inventory $Q_j$. Then demand realizes and inventory is used to satisfy customer orders. Both demand and inventory decisions are observable to both firms. If one of the firms, say firm j, is short of inventory and the other firm, say firm i, has surplus, then firm j requests $q_{ij} = \min\{D_j - Q_j, Q_i - D_i\}$ from firm i. After $q_{ij}$ units of inventory is transhipped to firm j, they are used to satisfy remaining customer demand. If there is still any remaining demand at firm j, then a random fraction $A$ of the remaining demand will switch to firm i, which we label as “switching demand". We assume that $A$ has cdf $F(.)$, pdf
f(.) and support [0, 1], and is independent of \(D_i\) and \(D_j\). \(F\) is twice differentiable and strictly increasing. Firm i then uses its inventory available at this time to satisfy these switching customers.

Firm i pays unit cost \(c_i\) for each unit of inventory purchased before demand realizes, and receives revenue \(r_i\) for each unit sold to its own customers or to customers switching from firm j. Firm i receives \(p_{ij}\) from firm j for each unit of inventory transhipped from firm i to firm j, and pays \(\tau_{ij}\) per unit as transportation cost. Any inventory left at the end of the period has salvage value \(s_i\) per unit. For each unit of unsatisfied local demand, firm i incurs penalty cost \(l_i\). We assume that there is no penalty cost charged if firm i cannot satisfy switching demand. We believe that this assumption is reasonable since customers usually feel more disappointed with their retailer of first choice. We make the following assumptions:

1. \(p_{ij} > \tau_{ij} + s_i\).
2. \(p_{ij} < r_i + \tau_{ij}\).
3. \(p_{ij} < r_j + l_j\).
4. \(r_i > c_i, c_i < c_j + \tau_{jjis}, s_i < s_j + \tau_{jjis}, \) and \(r_i < r_j + \tau_{jji}\).

OPTIMAL DECISIONS UNDER CENTRAL CONTROL

In this section, we consider the situation when the decisions of the two firms are made under central coordination to maximize total profit. We first examine the transhipment decisions after demand at both firms are observed and satisfied using available inventory, then we examine the optimal inventory purchasing decisions. Assume now firm i has inventory surplus and firm j has inventory shortage. We make a decision on the amount of inventory \(q_{ij}\) to be transhipped from i to j in order to maximize total profit \(\pi_t\). Define \(x^+ = \min(x, 0)\).

**Lemma 1**

When \(D_i + D_j \leq Q_i + Q_j\), if \(r_j - \tau_{ij} + l_j - r_i EA - s_i (1-EA) \geq 0\), then \(q_{ij} = D_j - Q_j\). Otherwise \(q_{ij} = 0\).

Lemma 1 indicates that when surplus at firm i exceeds shortage at firm j, the optimal decision is either transhipping all needed inventory, or not transhipping anything at all. Because the latter scenario of never transhipping anything is not of interest for this paper, in the subsequent discussion we make the following assumption (A1): \(r_j - \tau_{ij} + l_j - r_i EA - s_i (1-EA) \geq 0\).

When shortage at firm j exceeds surplus at firm i, depending on revenue and cost parameters, it might be more profitable to reserve some inventories at firm i anticipating that switching demand. Therefore an inventory rationing decision is needed. Let \(z_{it} = (Q_i - D_i - q_{ij})/(D_j - Q_j - q_{ij})\), which represents the ratio of firm i’s reserved inventory over the potential switching demand from firm j. Then the transhipment decision is equivalent to deciding \(z_{it}\), i.e., to reserve how much percentage of the potential switching demand as inventory rather than shipping to firm j. For each \(z_{it}\), \(q_{ij} = [Q_i - D_i - z_{it} (D_j - Q_j)]/(1-z_{it})\).

**Lemma 2**

If \(D_i + D_j > Q_i + Q_j\), let \(z_{it}^*\) be the solution of \(\int_0^{z_{it}^*} (1-a) f(a) da = (r_i + \tau_{ij} - l_j - r_i)/(r_i - s_i)\). Then if \((Q_i - D_i)/(D_j - Q_j) \leq z_{it}^*\), the optimal \(q_{ij} = 0\). Otherwise the optimal \(z_{it} = z_{it}^*\).

It is interesting to notice that \(z_{it}^*\) is independent of \(D_i, D_j, Q_i, Q_j\). Therefore, we should always try to keep the ratio of reserved inventory to potential demand as a constant. For example, if \(z_{it}^* = 0.2\), we should tranship some inventory such that the remaining inventory at firm i is 20% of the total potential switching demand. However, when inventory surplus at firm i is too small compared to shortage in firm j, it is possible that the remaining inventory is less than 20% of the total potential switching demand even without any transhipment. In this circumstance no transhipment should be arranged. Note that this transhipment decision is essentially a Newsvendor-type inventory decision. Therefore \(z_{it}^*\) depends on the
distribution of switching fraction \(A\), as well as revenue and cost parameters. The following corollary summarizes these effects, which are intuitive.

**COROLLARY 1** Given \(D_i, D_j, Q_i, Q_j\), the optimal transhipment quantity \(q_{ij}\) is non-increasing in \(r_i, \tau_{ij}, s_i\) and non-decreasing in \(l_j, r_j\).

The optimal transshipment decisions can also be illustrated by Figure 1, with regions where only partial transhipment should be arranged, and regions where no transhipment should be arranged at all, even when firm \(j\) faces shortage and firm \(i\) has surplus inventory.

Next we consider the inventory purchasing decisions before demand realization. Under central coordination, we choose \(Q_i, Q_j\) to maximize total expected profit of the two firms. We can show that under certain conditions there exist a unique optimal pair of inventory decisions.

**THEOREM 1** If \(g_i(.)\) and \(g_j(.)\) are non-increasing and all the parameters are symmetric, then \(E \pi\) is jointly concave in \((Q_i, Q_j)\). There exists a unique pair of \((Q_i^*, Q_j^*)\) that maximizes expected total profit.

The condition of non-increasing density functions holds when demand follows uniform distribution or exponential distribution. Note that we do not impose any condition on the density function of the switching fraction \(f(.)\).

**EQUILIBRIUM DECISIONS UNDER LOCAL CONTROL**

In this section, we consider the case where a central coordination scheme to coordinate the inventory and transhipment decisions for the two firms does not exist. Therefore each firm acts on its own interest. We find that the policy structure bears great similarity to the centralized case.

**LEMMA 3** When \(D_i + D_j \leq Q_i + Q_j\), if \(p_{ij} > \tau_{ij} + r_i EA + s_i (1-EA)\), then \(q_{ij} = D_j - Q_j\). Otherwise \(q_{ij} = 0\).

Similar to the centralized case, when total inventory exceeds total demand, either all shortage demand is fulfilled by transhipped inventory, or no transhipment takes place at all. However, the criterion is slightly different. For each unit of transhipped product, firm \(i\) receives \(p_{ij}\), and pays \(\tau_{ij}\) as shipping cost. Other than the shipping cost, firm \(i\) also incurs an opportunity cost since the expected revenue if otherwise reserved for switching demand is \(r_i EA + s_i (1-EA)\). Therefore, firm \(i\) does not tranship any inventory is \(p_{ij}\) is less than the total expected marginal cost \(\tau_{ij} + r_i EA + s_i (1-EA)\).

When total inventory is less than total demand, we define the same \(z_i = (Q_i - D_i - q_{ij}) / (D_j - Q_j - q_{ij})\), which represents the ratio of firm \(i\)'s reserved inventory over the potential switching demand from firm \(j\).

**LEMMA 4** If \(D_i + D_j > Q_i + Q_j\), let \(z_i^*\) be the solution of \(\int_0^{z_i^*} (1-a) f(a) da = (r_i + \tau_{ij} - p_{ij}) / (r_i - s_i)\). Then if \((Q_i - D_i) / (D_j - Q_j) \leq z_i^*\), the optimal \(q_{ij} = 0\). Otherwise the optimal \(z_i = z_i^*\).

Also similar to the centralized case, firm \(i\) should always keep the same ratio of reserved inventory over potential switching customers. The ratio \(z_i^*\) is different from \(z_i^{int}\) under central coordination. Comparing these two ratios, we have the following observation:
**Lemma 5** For any given $Q_i$, $Q_j$, $D_i$, $D_j$, $z_i^*$, $z_j^*$ and $q_{ij} \leq q_{ij}^*$.

Lemma 5 indicates that less inventory is transhipped without central coordination than with central coordination. This is due to the fact that shortage at firm $j$ not only does not result in any penalty cost for firm $i$, but also generates more switching demand to firm $i$. Therefore, shortage at firm $j$ is to a certain degree desirable to firm $i$. If firm $i$ makes transshipment decisions to maximize its own profit, it would reserve more inventory and transship less to firm $j$ than if the two retailers are centrally coordinated.

A graphical representation of the transshipment decision is the same as Figure 1 except that all superscript t's are eliminated. Thus firm $i$ may arrange fully transshipment, partial transshipment, or no transshipment depending on the amount of surplus compared to shortage.

Next we consider the inventory purchasing decisions if no coordination scheme exists. For any inventory purchasing decisions $(Q_1, Q_2)$, the expected profit for firm $i$ is given by:

$$E \pi_i(Q_i, Q_j) = -c_i Q_i + \sum E(\min(D_i, Q_i)) + (p_{ij} - \tau_{ij}) E q_{ij} + r_i E(\min[(Q_i - D_i - q_{ij}), A(D_j - Q_i - q_{ij})]) + s_i E[(Q_i - D_i - q_{ij}) - A(D_j - Q_i - q_{ij})].$$

We also show that under certain conditions a unique Nash Equilibrium exists.

**Theorem 2** If $g_i(.)$ and $g_j(.)$ are non-increasing functions, then there exists a unique Nash Equilibrium $(Q_i^e, Q_j^e)$.

**Lemma 6** There does not exist a coordination transshipment price $p_{ij}$ such that $\tau_{ij} + r_i EA + s_i (1-EA) \leq p_{ij} \leq r_j + l_j$ and $\pi_i(p_{ij}) + \pi_j(p_{ij}) = \pi^e(p_{ij}).$

Rudi et al. [4] show that there always exists a coordination price such that centralized profit can be achieved under decentralized control. Their results are later revised by Hu et al. [3] to show that such a coordination price only exists when the equilibrium inventory purchasing decisions for both firms are either more than or less than newsvendor quantity. However, when both transshipment and customer switching are considered, we show that such a coordination price does not exist in any situation. Rather than using a linear pricing mechanism, a more complicated mechanism is necessary for coordination and is interesting for future research.

**Summary and Discussion**

In this paper, we consider a transshipment model with customer switching behavior. We consider both centralized and decentralized situations and study the inventory purchasing and transshipment decisions. If a firm with surplus inventory receives transshipment request from another firm with stockout, it may ship only part of the requested amount while reserving the rest in hand, since some of the unfulfilled demand at the stockout firm may switch to buy at the firm with surplus. Such strategic transshipment decisions also affect the inventory purchasing decisions before observing demand.

Our model leaves some interesting future research directions. First, how should the transshipment price be determined? Since the transshipment amount depends on price, a higher transshipment price results in less transhipped inventory, while a lower transshipment price results in more transhipped inventory. If
one of the firms sets the price, there is an optimal price in between. It is unclear how endogenous transhipment price affects firms' inventory decisions. Secondly, is there a mechanism to coordinate the supply chain? And thirdly, it is interesting to observe that when partial transhipment takes place, higher shortage at firm j results in less transhipment from firm i. Therefore, firm j may have an incentive to underreport shortage. If shortage is unobservable, how would the shortage firm release information optimally? and how would the firm with surplus respond to that? Is there a mechanism that results in truth telling? All these questions are intriguing and we leave them for future work.

REFERENCES


![Figure 1: Transhipment Decisions Under Central Coordination](image-url)