FORECASTING USING FUZZY MULTIPLE OBJECTIVE LINEAR PROGRAMMING

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ABSTRACT

The application of fuzzy approach models to forecast has been drawing a lot of attention. This study proposes a fuzzy approach to forecasting using a financial data set. The methodology used is multiple objective linear programming (MOLP). Selecting an individual forecast based upon a single objective may not make the best use of available information for a variety of reasons. Combined forecasts may provide a better fit with respect to a single objective than any individual forecast. We incorporate soft constraints into a mathematical programming approach to improve our forecasting accuracy. We compare the results of our approach with the preemptive MOLP approach. A financial example is used to illustrate the efficacy of the proposed forecasting methodology.

Keywords: MOLP, Linear programming, Fuzzy, Forecasting

INTRODUCTION

An important problem facing decision makers in business organizations is the forecasting of uncertain events. The importance of having accurate forecasts available for decision making is widely recognized at all levels of decision making and in all functional areas of business. Because of the importance of the problem, multiple forecasts are often prepared for the same event. Decision makers have to evaluate the multiple forecasts with respect to a single objective and select the forecast method which comes closest to satisfying the chosen objective, while the remaining forecasts are discarded. Selecting a single forecast based upon a single objective may not make the best use of available information for a variety of reasons. Although there is likely to be some overlap or redundancy of information among forecasts, discarded forecasts may contain information not available in the selected forecast. Combined forecasts may provide better fit with respect to a single objective than individual forecast. Even if an individual
forecast does provide a good fit with respect to a single objective, a combined forecast may provide a better fit with respect to multiple objectives.

The purpose of this study is to investigate fuzzy approach to a combination of several techniques for forecasting monthly sales to produce improved forecasts over those produced by more traditional single technique approaches. This paper compares the forecast provided by combining the traditional forecasting with a fuzzy approach using soft constraints. The methodology used for combining the forecasts is preemptive multiple objective linear programming (MOLP).

LITERATURE REVIEW

Combining forecasts introduced by Bates and Granger (1969) is often considered as a successful alternative to using just an individual forecasting method. Empirical results demonstrate that no single forecasting method can generate the best forecasts in all situations and the relative accuracy of the different models varies with the origin/destination pairs and the lengths of the forecasting horizons [11]. Previous studies have shown that composite forecasting is useful in predicting variables of interest such as sales, corporate earnings per share, tourists inflow, etc [3] [10] [7] [12]. Wong et al. (2007) show that forecast combination can improve forecasting accuracy and considerably reduce the risk of forecasting failure. They conclude that combined forecasts can be preferred to single model forecasts in many practical situations. In a recent study, Hibon and Evgeniou (2005) propose a simple model-selection criterion to select among forecasts. Their results indicate that the advantage of combining forecasts is not only better results but also that it is less risky in practice to combine forecasts than to select an individual forecasting method.

The first application of forecasting using fuzzy set theory to our knowledge appeared in [4]. The author applied fuzzy concepts on the computer simulation for power demand forecasting and loading of power systems. Since this initial research, the Interest in fuzzy forecasting has grown considerably. Shnaider and Kandel (1989) develop a computerized forecasting system to forecast corporate income tax revenue for the state of Florida. Song and Chissom (1993) provide a theoretic framework for fuzzy time series modeling.

THE MODELS

Multiple Objectives Linear Programming
Preemptive multiple objective linear programming (MOLP) techniques are used to generate efficient combined forecasts. The forecasting techniques utilized in the study are exponential smoothing, multiple regression, and harmonic smoothing. A basic MOLP model for combining forecasts can be formulated as follows. The decision variables in the model are defined as the weights to be assigned to each forecast:
\( W_j \) = The weight assigned to forecast \( j \), \( j = 1, 2, \ldots, n \)

The coefficients of the model are the actual observed values and the forecasted values for each of the forecasts in each of the time periods considered:

\[ A_i = \text{The actual observed value in time period } i, \quad i = 1, 2, \ldots, m \]

\[ F_{ij} = \text{The forecasted value by forecast } j \text{ in time period } i, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \]

The constraints of the model take the following form:

\[
\sum_{j=1}^{n} F_{ij} W_j + d_i^- + d_i^+ = A_i \quad i = 1, 2, \ldots, m \tag{1}
\]

Where,
\[
d_i^- = \text{The underachievement by the combined forecast of the observed value in time period } i,
\]
\[
d_i^+ = \text{The overachievement by the combined forecast of the observed value in time period } i,
\]

and
\[
\sum_{j=1}^{n} W_j = 1 \tag{2}
\]

\[
W_j \geq 0, \quad j = 1, 2, \ldots, n
\]

\[
d_i^-, d_i^+ \geq 0, \quad i = 1, 2, \ldots, m
\]

In each time period, the weighted sum of the forecasted values plus or minus an error term must equal the actual observed value. The objectives of the model are expressed in terms of the underachievement and overachievement variables:

Minimize:
\[
Z = \left[ Z_1(\bar{d}^-, \bar{d}^+), \ Z_2(\bar{d}^-, \bar{d}^+), \ldots, \ Z_k(\bar{d}^-, \bar{d}^+) \right]
\]

Where
\[
\bar{d}^- = (d_1^-, d_2^-, \ldots, d_m^-), \quad \bar{d}^+ = (d_1^+, d_2^+, \ldots, d_m^+)
\]

The preemptive MOLP model objectives as formulated in (3) represent alternative measures of forecast error or accuracy. Alternative accuracy objectives allow decision makers to emphasize their preferences for the form and occurrence of forecast error. Two examples of such alternative measures are: (1) the minimization of total forecast error over all time periods, and (2) the minimization of the maximum forecast error in any individual time period. A third measure of error, which could be stated as an objective, is the minimization of forecast error in the most recent time periods. Thus, the overall MOLP model for combining forecasts consists of minimizing (3) subject to (1)-(2).

The associated objective functions are:
\[ Z_i = \sum_{i=1}^{30} (d_i^- + d_i^+) \]

\[ Z_2 = \sum_{i=1}^{30} d_i^+ \]

\[ Z_3 = \sum_{i=25}^{30} (d_i^- + d_i^+) \]

**Fuzzy Approach**

The fuzzy goal programming (FGP) model enlarges the feasible region by fuzzifying the constraints of the model with given tolerance values. Here, we use combination soft and crisp constraints instead of only crisp constraints in the MOLP. The fuzzy approach formulation is as follows [5]:

Minimize:  
\[ Z = \left[ Z_1(d^-_i, d^+_i), Z_2(d^-_i, d^+_i), \ldots, Z_k(d^-_i, d^+_i) \right] \]

Subject to:  
\[ \sum_{j=1}^{n} \tilde{F}_{ij} W_j + d_i^- + d_i^+ = \tilde{A}_i \quad i = 1, 2, \ldots, m \]

and  
\[ \sum_{j=1}^{n} W_j = 1, \quad W_j \geq 0, \quad j = 1, 2, \ldots, n \]

\[ d_i^-, d_i^+ \geq 0, \quad i = 1, 2, \ldots, m \]

Where \( \tilde{F}_{ij}, \tilde{A}_i, i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \) are fuzzy coefficients in terms of fuzzy sets.

**RESULTS**

<table>
<thead>
<tr>
<th></th>
<th>( Z_1 )</th>
<th>( Z_2 )</th>
<th>( Z_3 )</th>
<th>( W_1 )</th>
<th>( W_2 )</th>
<th>( W_3 )</th>
<th>( Z_1 )</th>
<th>( Z_2 )</th>
<th>( Z_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Forecast Error</td>
<td>414.89</td>
<td>232.82</td>
<td>123.04</td>
<td>0.60</td>
<td>0.09</td>
<td>0.31</td>
<td>108.25</td>
<td>108.25</td>
<td>44.01</td>
</tr>
</tbody>
</table>

**Table 1. Results**

Tables 1. Shows the results of our models. Several observations can be made based on the above results. First, combining different forecasting methods helps improve the level of one or more
objectives without worsening the level of any other objective. Second, using similar weights as in MOLP, the preemptive fuzzy approach give remarkably improved results.

REFERENCES


